ASYMPTOTIC AND EXPERIMENTAL ANALYSIS OF THE NONSPECULAR REFLECTION OF FOCUSING ULTRASONIC BEAMS FROM FLUID-SOLID INTERFACE

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INTRODUCTION

The interest of the use of ultrasonic focusing beams for NDT applications, particularly in the case of the reflection acoustic microscope, has led to the necessity of the study of the reflection of focusing beams.

In 1985, H.L.Bertoni et al.[1], studied the reflection of convergent beams on a liquid-solid interface in the Rayleigh incidence using the hypothesis of a well collimated beam. They obtained the position of the focal point of the reflected beam and forecast a lateral and axial displacement. This model has a number of advantages (simplicity and amenability to analytical solution), but it is difficult to apply in the form proposed by the authors to beams having more pronounced convergence, and an angle of incidence different from the Rayleigh angle.

In 1986, P.B.Nagy et al.[2], verified the axial displacement by means of Schlieren photography.

In this paper, we extend the previous theories in the following sense : (i) we introduced the notion of the caustic of the acoustic beam. (ii) An asymptotic evaluation of the reflected field was obtained by means of the stationary phase method applied to the Fourier representations. This allows one to explore the reflected pressure in a region which is not limited to the interface alone, and thus obtain a spatial knowledge of the reflected beam. (iii) A distortion of the caustic of the reflected beam was observed in the neighbourhood of the Rayleigh angle of incidence, including a lateral and axial displacement of the focal point of the beam. At the Rayleigh incidence, the lateral displacement is maximum; although the axial one is not nil, in comparison to the length of the focal spot it is negligeable and thus experimentally not detectable for this particular incidence. (iv) The nonspecular reflection of a focusing beam due to the existence of the Rayleigh wave (singularity on the real axis), occurs for any angle of incidence. For incidences near the Rayleigh angle, a part of the caustic, the acoustic axis, and the focal point are displaced; for another incidence, a

Acoustical Imaging, Volume 19
Plenum Press, New York 1992

different part of the reflected beam would be modified. (v) Moreover, we verify experimentally both lateral and axial displacements of the focalisation point by means of a cartography of the reflected beam.

MODELISATION OF THE REFLECTED FIELD

In order to describe the nonspecular reflection of focusing acoustic beams onto a plane liquid-solid interface, we assume that the incident field is given by a Gaussian distribution of the normal velocity along the plane of the emitter :

(1)
$$v_n(x_i,0) = V_0\, e^{-\left(\frac{x_i}{a}\right)^2}\, e^{\left(-ik \sin\theta_0 \frac{x_i^2}{a}\right)}\, e^{(-i\omega t)}$$

θ_0 : focalisation angle

and that the characteristic width of the beam, a, is large compared to the emission wave-length, k, (short wave hypothesis, ka>>1).

Let us consider the configuration of the figure 1.

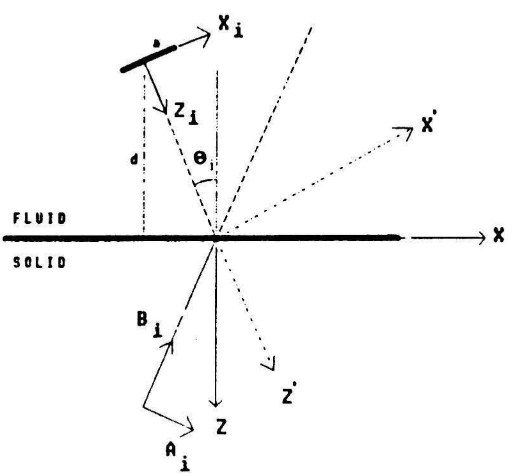

Figure 1. Coordonates definition

The reflected pressure field can be described by means of a Fourier representation [3].

(2)
$$P_{ref}(x',z') = C \int_{-\infty}^{+\infty} R(k_{x'})\; F(k_{x'})\; e^{i(ka) f_r(k_{x'})}\, dk_{x'}$$

where $R(k_{x'})$ is the plane-wave reflection coefficient.

The reflection coefficient has one pole in the complex $k_{x'}$-plane, where $k_{R'}=\sin(\theta_I-\theta_R)$ is denoted the real part of this pole (θ_R is the Rayleigh critical angle). For the special case of the Rayleigh incidence, the pole becomes purely imaginary. Moreover, it is always possible to develop the reflection coefficient in phase-modulus and regrouping the phase with the function f_r.
In the neighbourhood of the Rayleigh pole (which is a singularity in the complex plane), the modulus of the reflection coefficient tends towards infinity (when $k_{x'}\to k_{p'}$ we have $|R|\to+\infty$), whereas the phase is regular.
In the neighbourhood of the Rayleigh angle of incidence (which is a singularity on the real axis), the phase of the reflection coefficient varies rapidly, whereas the modulus remains regular.

By applying the stationary point method, the asymptotic expansion of the integral (2) is given by :

$$(3) \qquad P_{ref}(\bar{x}',\bar{z}') = \rho c V_o \sum_{i=1}^{n} \frac{R(\gamma_i)}{\sqrt{-2\sin\theta_o\, f''(\gamma_i)}} e^{-i\frac{\pi}{2}} \frac{e^{\left[\left(\frac{\gamma_i^2}{4\sin^2\theta_o}\right) + i(ka)\frac{\gamma_i^2}{4\sin\theta_o}\right]}}{\sqrt{1-\gamma_i^2}} e^{i(ka)\left(\gamma_i \bar{A}_i + \sqrt{1-\gamma_i^2}\,\bar{B}_i\right)} + O\left(\frac{1}{ka}\right)$$

where : γ_i are denoted the stationary points (or saddle-points) of the function f_r. (n=3 inside the focalisation zone, and n=1 outside this zone).
The general reflected caustic is thus given by :

$$(4) \qquad \left(\bar{A}_i + \frac{1}{n}\sum_{i=1}^{n} \frac{\varphi'(\gamma_i) - \gamma_i\,\varphi''(\gamma_i)}{ka}\right)^{2/3} + \bar{B}_i^{\,2/3} = \left(\frac{1}{2\sin\theta_o} + \frac{1}{n}\sum_{i=1}^{n}\frac{\varphi''(\gamma_i)}{ka}\right)^{2/3}$$

φ', φ'' : first and second derivatives of R.

In the case of an incidence near the Rayleigh angle, following the expression (4) and expanding γ_i about zero, we can deduce the modified part of this caustic and thus the position of the reflected focal point. By comparing this position with those of the specular focal point, we deduce the lateral, L, and axial, Ax, displacements of the focal point :

$$(5) \qquad L = -\frac{\varphi'(0)}{ka} = -\frac{\tilde{\varphi}'(\bar{k}_I)\cos\theta_I}{ka}$$

$$(6) \qquad Ax = \frac{\varphi''(0)}{ka} = \frac{\tilde{\varphi}''(\bar{k}_I)\cos^2\theta_I - \tilde{\varphi}'(\bar{k}_I)\sin\theta_I}{ka}$$

where $k_I=\sin\theta_I$

We must note that if at least one saddle-point remains in the neighbourhood of $k_{R'}$, in spite of an incidence far from the Rayleigh angle, there are points in the physical space for which the reflected field would be modified in comparison to values deduced from geometrical acoustics.

EXPERIMENTAL VERIFICATION OF THE LATERAL AND AXIAL DISPLACEMENTS OF THE REFLECTED FOCAL POINT

The experimental device contains a water tank, at the bottom of which a piece of aluminium, shaped according to the schema in figure 2, is immersed.

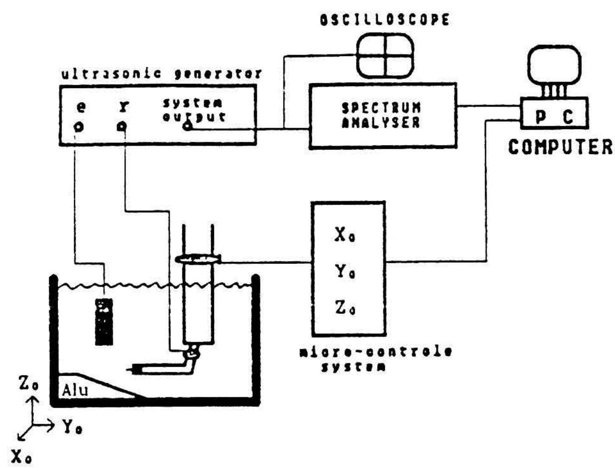

Figure 2. Configuration of the experimental device

The gradient of the piece of aluminium denoted θ_I, corresponds to the angle of incidence of the beam. We used four pieces of aluminium with different slopes :
θ_I=28.5° , θ_I=32.5° , θ_I=38° , $\theta_I=\theta_R$=30.5° where θ_R : is the Rayleigh critical angle for aluminium.

The emission part contains a 5MHz focusing transducer.

The reception part contains a miniature probe. We use this probe as a wide-band point receiver to investigate the propagation in water of short pulses from the emitter transducer. This transducer-receiver is linked to a micro-control system controled by PC computer. Thus the miniature probe can be moved in three directions : X_O , Y_O , Z_O. Automatic scanning enables the probe to carry out a displacement consisting of one movement following Yo thence Zo, in such a way that the global displacement remains parallel to the surface of the piece of aluminium.

The signal received is filtered through a "high pass" filter of 300 KHz, thence digitalized by means of a spectrum analyser. Once digitalized, the signal is windowed on 2048 points, then saved in the form of a file on the hard disc.
During the displacement of the receptor-probe we temporize, and we stock the digitalization of the preceding point.
This experimental process presents two advantages for the accuracy of the results :
a) The beam verticality in respect of the bottom of the tank being ensured, the shape of the aluminium pieces enables us to determine the angle of incidence of the beam with great precision.
b) The digitalization being carried out once the captor is fixed, the signal may be averaged out.

To each point in the plane (x,z) containing the reflected beam corresponds a time signal. In order to determine the focal point of the reflected beam and to evaluate the lateral and axial shifts, we carry out successively a Fast Fourier Transform thence a computation of the amplitude of the signal at the 5MHz emitting frequency.
A softwork enables us to go from the initial file of three variables (two spatial variables : x , z and time) and one parameter (the amplitude of the signal depending on time), to an output file of two variables (spatials : x and z) and one parameter (amplitude of the signal at 5MHz).
The figures 3 and 4 represent the trace by isolevels of the output file, for different angles of incidence. Thus we obtain the cartography of the reflected field [4].

Figure 3 represents the reflected field for an angle of incidence of 38° (far from the Rayleigh angle). We deduce no lateral or axial shift; this result corresponds to that of the geometric acoustics.

Figure 4 represents the reflected field for an angle of incidence of 30.5°, which corresponds to the Rayleigh angle. We deduce no axial shift and a lateral one of about 5.5 mm.

For an angle of incidence of 28.5° and 32.5° (in the neighbourhood of the ayleigh angle) we deduced a lateral shift of about 3mm, and an axial one of -43mm and 45mm respectively.

CONCLUSION

In this paper we have studied the structure of the reflected field when focusing Gaussian beam is incident onto a liquid-solid interface. An analytical expression of the reflected pressure field was obtained by means of the asymptotic method of stationary phase based on the short wave hypothesis; this expression is valid for any incidence. A modification of the structure of the reflected focusing beam in

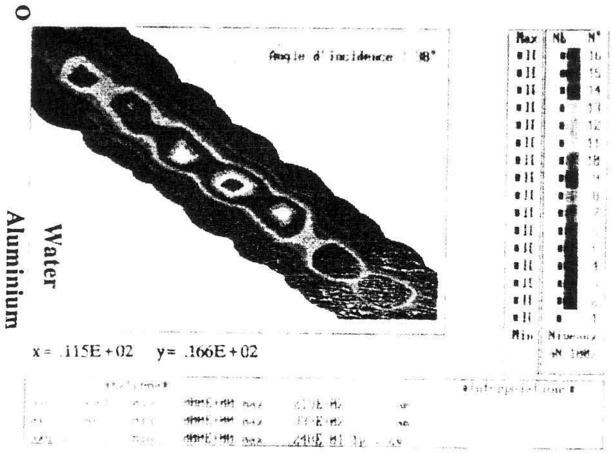

Figure 3

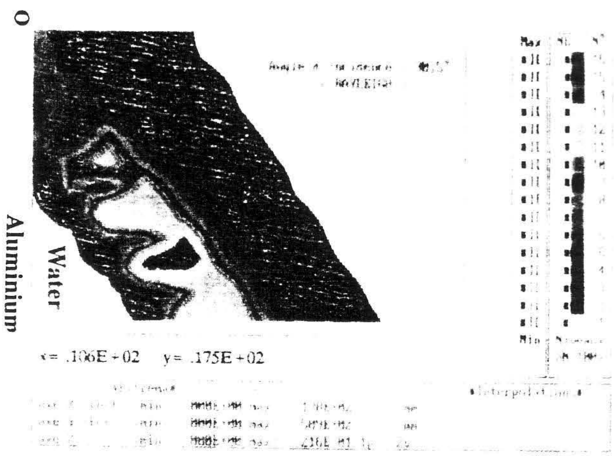

Figure 4

comparison to geometric acoustics was observed for any incidence; in particular, for the Rayleigh incidence, the nonspecular phenomena concern a part of the reflected caustic including the focal point. Simple expressions of the axial and lateral displacements of the focal point were obtained, and led to a numeric quantification. These results were verified experimentaly by means of a cartography of the reflected beam. In the case of the reflection of a focusing beam there is no reradiation of a leaky Rayleigh wave (responsible for the nonspecular reflection of a parallel beam[5]); here nonspecular phenomena are due to the abrupt variation of the phase of the reflection coefficient in the neighbourhood of the Rayleigh angle.

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